

Refinement of the Structure of Potassium Heptafluoroniobate, K_2NbF_7 , from Neutron-Diffraction Data*

BY GEORGE M. BROWN AND LEWIS A. WALKER†

Chemistry Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee, U.S.A.

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The crystal structure of K_2NbF_7 , determined originally from two-dimensional X-ray data, has been refined by the method of least-squares with three-dimensional data. The refinement included adjustment of the scattering factors of niobium and potassium relative to the fluorine scattering factor. For potassium the new value $0.370 \pm 0.004 \text{ cm}^{-12}$ was found. Effects of extinction were minimized by omitting from the final refinement a number of the observations, as suggested by systematic analysis of the quality of agreement between calculated and observed F^2 values as a function of observed intensity. Shifts of atomic positions by as much as 0.15 \AA occurred in the refinement, but the general description of the structure, including the description of the NbF_7^{2-} ion, remains unchanged. The standard errors of the coordinates are about 0.001 \AA for the niobium atom and 0.002 to 0.003 \AA for the fluorine atoms and the potassium ions. Analysis of the anisotropic thermal parameters shows that the NbF_7^{2-} ion is a nearly ideal case for use of the 'riding' model in calculating bond-length corrections, and the Nb-F bond lengths have been corrected accordingly. Exceptionally large vibrational displacements of two of the fluorine atoms seem plausible in view of the packing in the crystal.

Introduction

Our interest in the crystal structure of potassium heptafluoroniobate, K_2NbF_7 , arose in connection with recent work (Keller, 1963) in this laboratory on the Raman and infrared spectroscopy of complex ions of niobium(V) in hydrofluoric acid solutions. The original determination of the K_2NbF_7 structure (Hoard, 1939), from two zones of X-ray reflections showed that the NbF_7^{2-} ion has approximately the symmetry C_{2v} . The polyhedron of fluorine atoms was visualized as derived from an NbF_6 group in the form of a trigonal prism by the addition of a seventh fluorine atom through the center of one square face, followed by some distortion. Although there was no particular reason to question the essential correctness of this structure, we thought it desirable to confirm the coordination about niobium and to refine the structure from three-dimensional neutron data. For K_2NbF_7 it is advantageous to employ neutron analysis, rather than X-ray analysis, for two reasons. First, the narrow range of values of the neutron scattering factors of K, Nb, and F nuclei ensures that the structural parameters of all atoms will be determined with nearly the same degree of precision. Second, the low absorption coefficient of K_2NbF_7 for thermal neutrons makes it possible to obtain intensity data virtually free from absorption errors.

Experimental data

Unit-cell parameters

The unit-cell parameters (Table 1) were redetermined from measurements of angles of Bragg X-ray reflect-

ions from a small crystal specimen. The measurements were made on a Picker diffractometer fitted with a General Electric single-crystal orienter. The translations a , b , c were determined from high-order reflections $h00$, $0k0$, and $00l$ for which the $Cu K\alpha_1 - \alpha_2$ doublet was resolved. The monoclinic angle β was determined directly by measurement of the angles between the reciprocal axes a^* and c^* . The new cell parameters are close to those of Hoard, but more precisely determined.

Table 1. *Unit-cell parameters of K_2NbF_7*

	Hoard*	This research†
a	5.85 \AA	$5.846 \pm 0.003 \text{ \AA}$
b	12.67	12.693 ± 0.006
c	8.50	8.515 ± 0.004
β	90°	$90.0^\circ \pm 0.1^\circ$
Z		4
Space group		$P2_1/c$

* 'Estimated limits of error' in translations are 0.02 to 0.03 \AA .

† The wavelengths assumed for $Cu K\alpha_1$ and $Cu K\alpha_2$ radiation were 1.54050 and 1.54434 \AA . The indicated error for each parameter is the root-mean-square deviation from the mean of several observations.

Intensity data

A crystal specimen weighing 18.5 mg was selected from a bottle of reagent-grade material purchased from the Kawecki Chemical Company. Its over-all dimensions were about 1.3 , 1.9 , and 3.6 mm . Preliminary X-ray precession photography established that it was a single crystal. The crystal was mounted with the a axis (3.6 mm dimension) parallel to the spindle axis, then sealed inside a thin-walled tube of quartz, for protection from moisture.

Intensity data were recorded by the $\theta-2\theta$ step-scan method with the Oak Ridge automatic neutron dif-

* Research sponsored by the U.S. Atomic Energy Commission under contract with the Union Carbide Corporation.

† Summer Research Participant, 1963. Present address: Department of Chemistry, Goucher College, Towson, Maryland.

fractometer (Busing, Smith, Peterson & Levy, 1964).¹ Counts were recorded at 0.1° intervals over a 2θ range of 5° centered around the expected peak position of each reflection. For each point in the scan counts were registered during the period (~ 8 seconds) required for a counter monitoring the primary beam to register a preset fixed count. The neutron wavelength was 1.078 \AA . For the [100] and [001] zones, data were recorded out to 111° in 2θ , the limit of the accessible range. Subsequently the general reflections hkl were recorded in successive concentric shells in reciprocal space to the 2θ limit of $\sim 85^\circ$, beyond which it appeared that the number of data with intensities above background would be small. Two reflections of high intensity, 400 and 060, were recorded at regular intervals, after every 50 observations, as a check on the performance of the instrument and the stability of the crystal. A number of other reflections were also observed more than once. In all, 1752 observations were made, corresponding to 1355 independent reflections.

Preliminary data processing

In a series of calculations on the CDC 1604 and IBM 7090 computers the raw data on punched paper tape were converted into a set of least-squares data on cards. After a preliminary step* in which the net count N and the background count B under each peak were computed, the calculations* implied by the following equations were performed:

$$F_o^2 = A_c S L N \quad (1)$$

$$\sigma^2 F^2 = (A_c S L)^2 [N + (1+r)B] \quad (2)$$

where F_o^2 is the observed value of the square of the structure factor; $\sigma^2 F^2$ is the variance of F_o^2 from the counting statistics; S is a scale factor derived from the weight of the crystal specimen and measurements of the intensity of reflection 400 from a standard crystal of sodium chloride; A_c is the correction factor for absorption; L is the Lorentz factor, $\sin 2\theta$; and r is the ratio of the number of points under the peak to the number of neighboring exterior points used in establishing the background count.

In the calculation of the absorption factors A_c (Wehe, Busing & Levy, 1962) the crystal specimen was described by specifying the equations of 13 surface planes. The value 0.243 cm^{-1} , calculated from tabulated neutron cross sections (S.W. Peterson, unpublished) was used for the linear absorption coefficient. The largest factor A_c was 1.044.

For those reflections for which two or more observations were made the average F_o^2 values and their appropriate variances were computed, using weights equal to the reciprocals of the variances of the individual observations. Negative values of F_o^2 were replaced by zero values. Finally, an empirical correction

term $(0.03 F_o^2)^2$ was added to the variance $\sigma^2 F^2$ of the average F_o^2 for each independent reflection. The extra term makes allowance for possible instability in the instrument, errors in corrections for absorption, deficiencies of the structure model, and so forth (see Peterson & Levy, 1957).

The entire procedure followed typically in neutron-diffraction analyses in this laboratory is presented in more detail in an earlier paper on problems and procedures in data collection (Brown & Levy, 1964). The same paper presents a statistical analysis of consistency among the many replicate measurements of the two standard reflections, 400 and 060, of K_2NbF_7 .

Refinement

Refinement was carried out by the method of least squares, using various versions of the full-matrix least-squares program of Busing, Martin & Levy (1962). The function minimized was $\Sigma w_{F^2} (F_o^2 - F_c^2)^2$, where F_o^2 and F_c^2 are the observed and calculated structure-factor squares, and w_{F^2} is the weight of the observation F_o^2 , given by $w_{F^2} = 1/\sigma^2 F^2$.

When the 288 data of the [100] and [001] zones had been recorded and processed, a least-squares calculation was made to establish preliminary values of the isotropic thermal parameters B of the various atoms, since the work of Hoard did not provide any thermal parameters. In three cycles of least-squares adjustment, in which Hoard's atomic coordinates were held fixed, the values of B spread from the common assumed input value 1.0 \AA^2 through the range from 1.03 \AA^2 (for niobium) to 4.23 \AA^2 . The value of the discrepancy index

$$R_{F^2} = \Sigma |F_o^2 - F_c^2| / \Sigma F_o^2$$

was reduced from 0.57 to 0.21 (omitting 28 reflections of uncertain sign).

Fourier syntheses along [100] and [001] were computed with the use of the observed structure factors F_o with the signs of the structure factors F_c calculated after adjustment of the thermal parameters. The maps of nuclear scattering density were very similar to the electron-density maps of Hoard, except for the expected changes in relative peak heights. It was clear at this stage that the Hoard structure is basically correct.

The subsequent least-squares refinement on the three-dimensional data was generally straightforward. Special care was exercised, however, to evaluate the extent of extinction errors in the data and to minimize the effects of these errors. While refining the structural parameters, we were also concerned with the possibility of obtaining more reliable relative values of the scattering factors of potassium, niobium, and fluorine. Accordingly, we carried out six different refinements to convergence, as described in Table 2 and explained below. The convergence in every case was such that the final shift for each parameter was less than 0.02 times the corresponding standard error.

* The indicated calculations were performed with computer programs of W. R. Busing.

Soon after the three-dimensional refinement was started, comparison of the values F_o^2 and F_c^2 showed that a number of the values F_o^2 for reflections of high intensity were in error because of extinction. To estimate the extent of the errors, we examined plots of the functions $\langle F_o^2 \rangle / \langle F_c^2 \rangle$ and $\langle |F_o^2 - F_c^2| / \sigma_{F_o} \rangle$ against $\langle I_o \rangle$, where $I_o = F_o^2 / \sin 2\theta$ and the brackets $\langle \rangle$ denote an average value over a small range of I_o . In Fig. 1, curves (a) and (b) show the plots of these functions after refinement to convergence of the atomic coordinates and anisotropic thermal parameters* and the scale factor, when 1354 F_o^2 data** were included (refinement A in Table 2). The plots suggest that the F_o^2 values corresponding to I_o greater than about 40 (in units of 10^{-24} cm^2) are nearly all low by a few per cent because of extinction. When refinement was carried to convergence omitting the data corresponding to $I_o > 40$, the scale factor on F_o^2 decreased, and examination of the new fit of the data showed that even more reflections are affected by extinction. Thus, estimation of the extent and magnitude of extinction errors is necessarily an iterative process. In our final refinement (E) we have excluded 112 observations, all those with $I_o > 19.80$. The curves (c) and (d) of Fig. 1 show the general agreement of F_o^2 and F_c^2 values in this case. The choice of the final cut-off level of I_o is somewhat arbitrary; but, fortunately, it appears not to be critical. Thus, comparisons of entries B and D and of entries C and E of Table 2 show that the goodness of fit as measured by the values of σ_1 , the standard deviation of an observation of unit weight,† is insensitive to a change in

* The thermal parameters β_{ij} are the coefficients in the exponent of the temperature factor $\exp -[\beta_{11}h^2 + \beta_{22}k^2 + \beta_{33}l^2 + 2\beta_{12}hk + 2\beta_{13}hl + 2\beta_{23}kl]$.

** Reflection 342 was omitted from all refinements after it was noticed that the shape of its peak on the recorder chart was anomalous, probably because of an instrumental aberration.

† The quantity σ_1 is given by the expression $[\sum w_F^2(F_o^2 - F_c^2)/(n-p)]^{1/2}$, where n is the number of observations and p is the number of parameters adjusted. Unit value is expected for σ_1 at convergence when the observational errors are normally distributed and correctly estimated and the model is correct.

the cut-off level of I_o from 26.61 to 19.80. The scale factor is similarly insensitive to this change. Comparison of entries A, B, and D shows that the fit is distinctly poorer when all of the data are used in the refinement. It is of interest that the discrepancy factor, whether calculated on $|F|$ or on $|F|^2$, is a very unreliable index of quality of fit in these refinements.‡

The entries under F show approximately the quality of fit which might have been expected if there had been no perturbation from extinction. For this refinement the data corresponding to $I_o > 19.80$ were corrected by multiplication of each F_o^2 value by the reciprocal of the ordinate read from the curve of Fig. 1(c) at the appropriate I_o .

The shapes of the curves (a) and (c) of Fig. 1 are not precisely what one might expect. For one thing, it seems that our intensity measurements for the very weak reflections are on the average slightly high be-

‡ The fact that the discrepancy factor computed on $|F|$ can equal or exceed the factor computed on $|F|^2$ has been discussed by Brown (1966).

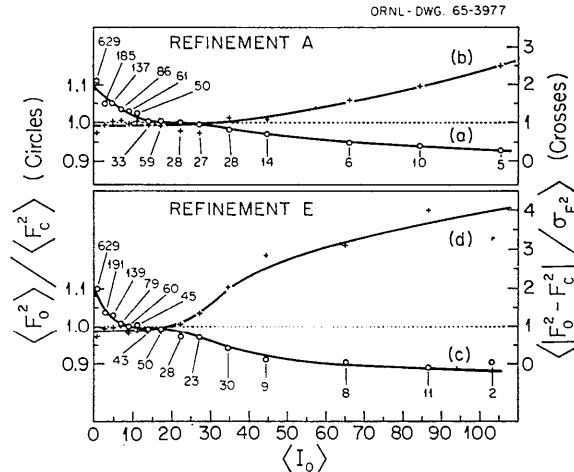


Fig. 1. Curves showing the agreement between the values F_o^2 and F_c^2 as a function of intensity (see text). The numbers attached to the data points are the numbers of reflections included in the ranges of intensity over which the averages were taken.

Table 2. Description of the six different refinements to convergence

In all six cases the scale factor on $|F_c|$ and the three coordinates and six anisotropic thermal parameters of each atom were adjusted. The scattering factor of fluorine was fixed at $0.55 \times 10^{-12} \text{ cm}$ throughout. The scattering factors of potassium and niobium were held fixed in refinements A, B, and D at $0.35 \times 10^{-12} \text{ cm}$ and $0.691 \times 10^{-12} \text{ cm}$ but were adjusted in C, E, and F. The values of the discrepancy indices in parentheses were computed from all 1355 data

Refinement	Number of data included	Scattering factors derived (units of 10^{-12} cm)	Scale factor derived	$(\sum F_o ^x - F_c ^x)/\sum F_o ^x$	σ_1
				$x=2$	$x=1$
A	1355	not adjusted	0.984 ± 0.003	0.081	0.090
B	1278	not adjusted	1.006 ± 0.003	0.095	0.103
C	($I_o \leq 26.61$)	$f_{Nb} = 0.695 \pm 0.005$	1.001 ± 0.004	(0.090)	(0.092)
D	($I_o \leq 26.61$)	$f_K = 0.370 \pm 0.004$	1.014 ± 0.004	0.094	0.103
E	($I_o \leq 19.80$)	not adjusted	1.009 ± 0.004	0.104	0.110
F	($I_o \leq 19.80$)	$f_{Nb} = 0.692 \pm 0.005$	1.005 ± 0.003	(0.094)	(0.093)
	(corr. data)	$f_K = 0.370 \pm 0.004$		0.103	0.109
		$f_{Nb} = 0.695 \pm 0.003$		(0.094)	(0.093)
		$f_K = 0.366 \pm 0.003$		0.070	—
					1.11

cause of some systematic errors of unknown origin, possibly effects of multiple reflection. Although curves (a) and (c) give a rather exaggerated impression of this effect, curves (b) and (d) show that the systematic error is not large in relation to the standard error of measurement. At high intensities the ratio $\langle F_o^2 \rangle / \langle F_c \rangle$ does not fall so abruptly from unit value as one would expect for an extinction effect. The detailed shape of the curve will be affected, however, by anisotropy in the extinction and, possibly, by some errors from other sources. In any case, we make use of the curve mainly to avoid the inclusion of questionable data in the refinement, not for any quantitative analysis of extinction effects.

The sets of atomic coordinates from the various refinements differ only slightly from each other. From refinements *B*, *C*, *D*, and *E*, the average ratio over all 30 coordinates of the range of coordinate variation to the corresponding coordinate standard error is about 0.3 and the extreme ratio is 0.7. Even when refinement *A* is included, the average and extreme ratios are only 0.6 and 1.2. Between *D* and *E*, which differ only as

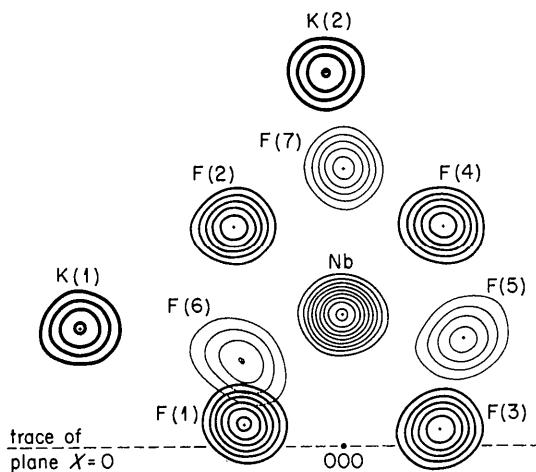


Fig. 2. Final composite three-dimensional Fourier map of the asymmetric unit of the K_2NbF_7 structure. The direction of view has direction cosines 0.0, -0.669, -0.743 with respect to the crystal axes a , b , c . The contour interval is 0.5 in units of 10^{-12} cm^{-3} . The lowest contour is at the 0.5 level.

regards adjustment of f_{Nb} and f_K , the average ratio is 0.2 and the extreme is 0.4.

In general, the thermal parameters are different only by similar small amounts from one refinement to another. The largest shifts noted are the increases of about two standard deviations in the β_{ii} parameters of the potassium ions from *B* to *C* and from *D* to *E*, which accompany the increase in the value of f_K . The inclusion in refinement *A* of the data affected by extinction is compensated for largely by a change in the scale factor, but partly by slight decreases in most of the β_{ii} .

For no single parameter is there a significant change in the standard error from one refinement to another. It is interesting, however, that the computed errors of the β_{ii} parameters of niobium and potassium are all 10 to 15% larger when f_{Nb} and f_K are included in the least-squares adjustment, reflecting the strong correlation between the scattering factor and the parameters β_{ii} of a given atom.

It would have been unnecessary to adjust the scattering factors f_{Nb} and f_K , as was done in refinements *C*, *E*, and *F*, if these parameters had already been precisely determined. However, the tabulated values (Bacon, 1963) of f_{Nb} , f_K , and f_F are certainly subject to revision. They were determined from powder data, along with the scattering factors of many other elements, by Shull & Wollan (1951), who claimed in general only an accuracy of 3 to 4%. Hughes & Harvey's (1955) estimates of the standard errors of the coherent scattering cross-sections correspond to standard errors of about 1.7, 3.5, and 4.0% in the values f_{Nb} , f_K , and f_F , respectively. Since scale factors are normally obtained in least-squares refinements of crystal structures with considerably smaller standard errors than these, it seemed to us desirable to attempt refinement of the relative values of the scattering factors. From our results (Table 2) the true value of f_K relative to the value 0.55 cm^{-12} for fluorine* appears to be $0.370 \pm 0.004 \text{ cm}^{-12}$, significantly different from the tabulated value 0.35 cm^{-12} . The value of f_{Nb} is not changed significantly. Although these

* The choice of the scattering factor of fluorine as reference is arbitrary.

Table 3. Final atomic coordinates and thermal parameters of the K_2NbF_7 crystal structure (from refinement *E*)

Atom	Fractional coordinates $\times 10^4$ (standard errors $\times 10^5$)			Thermal parameters $\times 10^5$ (standard errors $\times 10^5$)					
	x	y	z	β_{11}	β_{22}	β_{33}	β_{12}	β_{13}	β_{23}
Nb	2718(20)	1288(9)	2229(14)	799(36)	157(7)	376(15)	3(12)	34(17)	20(8)
K(1)	2397(46)	4404(19)	1833(31)	1699(82)	232(15)	698(37)	-42(27)	-37(41)	-40(18)
K(2)	7626(45)	2845(22)	4446(30)	1343(81)	424(18)	590(36)	-8(30)	-1(42)	-98(19)
F(1)	463(28)	2431(13)	1994(20)	1188(46)	275(10)	759(22)	-152(18)	38(27)	10(13)
F(2)	4573(31)	2567(14)	2114(24)	1198(47)	240(9)	1159(28)	116(18)	239(29)	99(14)
F(3)	448(30)	1072(14)	3880(21)	1326(50)	355(11)	673(22)	59(18)	-288(27)	48(13)
F(4)	4599(31)	1107(16)	4101(20)	1263(50)	542(13)	530(21)	81(20)	223(27)	137(14)
F(5)	2166(41)	-207(15)	2159(30)	3041(81)	228(10)	1682(40)	296(22)	-1043(47)	-198(16)
F(6)	1875(46)	1216(21)	63(25)	3675(90)	834(20)	633(24)	-949(35)	628(39)	-278(19)
F(7)	5572(31)	792(15)	1317(24)	1336(52)	407(12)	927(26)	-142(20)	-337(29)	15(14)

Table 4. Observed and calculated structure-factor squares

Reflections are identified by the indices h and k in the sub-headings and the index k under the main heading K . For each reflection $|F_c(hkl)|^2$, $|F_o(hkl)|^2$, and the standard error σ_{F^2} (SIG) are given (all in units of 10^{-26} cm^2), along with the sign of $F_c(hkl)$. A letter to the left of an index k indicates omission of the reflection in the final refinement, because of extinction error (E), an aberrant measurement (A), or a missing observation (M).

K	CALC	OBS	SIG	K	CALC	OBS	SIG	K	CALC	OBS	SIG	K	CALC	OBS	SIG	K	CALC	OBS	SIG	K	CALC	OBS	SIG					
***	0	K	0	***	0	K	0	***	0	K	0	***	0	K	0	***	0	K	0	***	0	K	0	***				
0	44650	-	-	4	287	271	50	13	903	921	55	7	468	538	40	4	223	175	38	7	363	376	34	0				
2	22	13	8	5	21	30	36	14	111	127	26	8	1178	1145	58	5	548	478	42	8	91	76	34	1				
4	863	49	10	6	10	20	30	15	1970	2068	76	9	107	91	47	6	70	50	38	9	65	130	41	2				
6	5200	442	146	E	3275	331	50	16	10	10	20	10	40	502	493	40	7	111	147	36	10	208	226	102	E			
E	6251	5691	176	8	397	321	50	17	159	186	42	11	176	152	36	8	90	161	37	11	157	157	69	4				
10	42	25	20	10	120	1229	60	19	91	91	41	12	107	54	50	9	1	0	39	12	779	158	65	5				
12	39	25	20	10	347	308	50	19	1	0	0	13	532	556	48	10	443	450	51	13	208	183	39	6				
14	377	343	26	11	72	86	38	14	242	324	44	14	281	353	43	14	281	353	43	7	3	15	54	+				
16	1040	1078	45	12	354	366	44	16	1	1	0	14	242	324	44	16	8	67	89	35	8	67	89	35	+			
18	189	158	30	15	5	15	5	15	15	15	5	15	15	15	5	15	15	15	15	15	15	15	15	+				
***	0	K	0	***	0	K	0	***	0	K	0	***	0	K	0	***	0	K	0	***	0	K	0	***				
0	55	156	41	E	1	587	569	21	2	127	145	12	E	0	2746	2402	78	1	167	198	36	0	216	268	17	***		
4	66	51	6	17	99	76	59	3	429	406	18	1	956	903	35	2	18	33	32	1	187	180	16	***				
E	2453	2255	176	E	1	2004	2000	88	4	201	205	15	E	2	1316	1212	44	5	65	109	39	E	2	2068	2454	79	***	
4	46	42	12	***	0	K	7	***	0	K	7	***	0	K	7	***	0	K	7	***	0	K	7	***				
E	5	388	376	19	E	1	194	65	56	8	534	537	28	E	7	104	104	56	4	602	582	43	E	5	1616	1707	50	***
7	4801	4389	137	E	2	154	65	56	10	152	130	19	E	7	117	1134	54	8	964	992	56	E	7	1759	1754	72	***	
9	1111	1042	61	4	340	311	50	11	50	50	50	11	50	50	50	8	15	15	42	8	30	42	42	+				
9	5	5	20	5	719	761	49	12	98	87	40	10	213	200	58	5	543	543	36	5	292	99	56	5				
10	59	81	93	6	193	215	37	13	537	547	55	11	15	15	0	***	1	K	9	***	11	196	226	39	7			
11	110	99	46	7	255	169	36	14	316	319	42	12	17	42	33	12	1697	1604	72	8	52	78	35	+				
12	0	54	0	8	285	265	50	15	0	0	40	13	356	328	50	1	22	30	35	13	261	308	39	9				
X	321	321	0	9	108	107	50	16	0	44	37	14	36	32	70	41	3	16	15	35	12	152	154	38	11			
14	465	421	43	IC	20	40	50	15	250	321	41	***	1	K	-1	***	0	100	185	36	***	2	K	3	***			
15	145	170	40	12	0	42	42	E	1	980	938	51	7	25	0	40	E	2	2596	2040	68	***	2	K	7	***		
16	20	32	40	12	0	42	42	E	2	134	152	12	E	1	2262	2058	76	6	11	60	38	E	1	35	34	16	1	
17	109	83	45	13	57	62	42	5	429	406	18	E	2	1374	1410	58	7	25	0	40	E	2	259	2040	68	***		
18	23	28	40	13	155	93	43	5	571	531	23	E	3	1760	1760	50	4	436	409	39	***	1	100	185	36	***		
19	192	87	42	13	58	61	42	5	538	515	15	E	6	3782	3472	113	1	64	89	35	***	6	240	248	28	3		
***	0	K	2	***	0	K	8	***	0	K	8	***	0	K	8	***	0	K	9	***	6	668	746	33	4			
E	0	2155	1989	62	8	21	28	50	E	0	1002	922	32	E	2	1345	1285	53	2	155	1516	69	E	2	4789	4310	135	***
1	1	1	1	0	12	0	30	9	70	70	44	0	10	455	466	66	3	118	118	30	E	2	158	204	82	***		
2	4	4	10	1	215	204	36	10	455	466	66	0	10	450	464	64	4	59	59	36	E	2	2168	2040	82	***		
E	4	2225	2077	68	3	1535	1582	69	12	64	50	55	10	389	352	41	5	150	137	30	9	0	8	77	36			
E	5	1300	1297	46	E	2	2484	2372	92	13	192	136	39	12	85	128	128	13	6	1066	1129	61	11	124	128	34	9	
7	1	193	190	19	5	0	34	14	15	69	67	41	15	53	73	50	15	433	446	44	15	527	546	44	7			
8	995	1008	41	7	83	104	50	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15				
9	54	89	22	8	19	19	50	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15			
E	10	1973	1911	80	9	787	736	53	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15		
E	11	3492	3252	118	10	369	399	46	E	0	1002	922	32	E	2	1345	1285	53	2	155	1516	69	E	2	4789	4310	135	***
12	241	188	37	11	449	459	49	1	242	275	15	3	116	139	20	3	748	540	49	4	59	59	36	4				
13	106	68	33	12	45	42	42	2	275	113	15	4	16	42	27	4	252	334	47	5	58	57	33	5				
14	175	152	34	13	249	292	50	E	3	1545	1545	50	1	152	152	50	1	152	152	50	6	152	152	50	6			
15	65	51	40	14	14	40	50	5	538	515	15	7	152	152	50	1	152	152	50	6	152	152	50	6				
16	899	893	59	15	85	198	40	6	538	515	15	8	152	152	50	1	152	152	50	6	152	152	50	6				
17	710	857	56	2	187	187	27	9	185	187	27	10	7	84	435	474	46	1	729	808	53	10	20	16	47	10		
18	686	741	56	3	107	107	39	15	31	72	72	12	7	747	775	50	15	873	878	53	10	20	16	47	10			
***	0	K	3	***	2	25	29	34	11	245	310	41	15	381	422	48	12	112	85	40	0	564	334	41	***			
1	19	5	10	4	230	259	50	15	148	148	46	15	172	179	19	1	171	171	35	15	146	158	40	14				
E	2	2934	2616	50	6	107	107	50	15	148	148	46	15	172	179	19	1	171	171	35	15	285	349	39	14			
E	4	2295	2100	71	7	750	726	54	15	604	604	53	E	0	8150	7078	219	1	183	179	80	0	5	923	909	53	***	
6	5	197	215	21	8	1	0	41	5	172	179	19	10	9	171	173	53	E	2	1048	953	32	0	5	923	909	53	***
7	1	20	16	4	1	9	138	50	6	171	171	31	9	10	21	50	15	E	2	1048	953	32	0	5	923	909	53	***
8	160	182	43	1	364	419	49	8	171	171	31	10	3	54	54	50	15	11	11	42	42	0	5	923	909	53	***	
9	91	92	41	2	2	66	39	3	9	167	149	25	12	279	279	45	15	15	15	15	15	15	15	15	15	15		
10	69	82	38	15	153	153	50	15	123	187	40	15	15	8	32	39	15	15	15	15	15	15	15	15	15	15		
11	147	145	34	1	158	181	57	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15		
12	310	301	38	3	8																							

Table 4 (cont.)

K	CALC	OBS	SIG	K	CALC	OBS	SIG	K	CALC	OBS	SIG	K	CALC	OBS	SIG	K	CALC	OBS	SIG	K	CALC	OBS	SIG							
***	3	K	0	***	3	K	4	***	3	K	-8	***	3	K	-5	***	4	K	-8	***	5	K	-4	***						
16	5	17	28	-	12	182	210	42	6	64	51	40	5	566	429	42	0	4	0	2638	239	92	0	44	98	57				
15	15	66	30	-	13	554	645	54	7	519	536	49	4	115	252	41	1	327	365	45	1	21	21	56						
16	1	14	3	-	1	14	17	3	1	3	5	5	1	52	45	1	2	65	92	30	2	12	22	56						
17	211	180	45	***	3	K	-4	***	3	K	9	***	6	63	27	56	3	678	689	52	3	45	96	57						
18	5	0	40	-	0	47	58	22	2	1586	1692	77	9	20	50	32	5	1501	1278	67	4	3	39	55						
***	3	K	1	***	1	58	58	22	1	717	657	52	8	50	32	54	5	28	0	43	5	21	292	46						
E	1127	1067	40	-	E	1820	1778	62	3	1	20	40	10	519	489	6	***	5	K	0	***	7	103	107	51					
E	1655	1717	40	-	E	2544	2385	91	5	476	450	50	12	638	750	57	0	85	92	30	8	1103	1086	61	9	491	526	56		
151	161	21	-	6	730	716	50	***	3	K	-9	***	***	4	K	-4	***	5	K	0	***	7	706	712	56					
5	1	27	26	-	7	269	367	45	1	118	92	41	0	99	161	41	2	105	91	35	***	6	K	3	***					
1303	1225	56	-	9	427	427	42	2	1527	1611	77	1	655	756	53	4	201	162	31	1	595	583	50	2	241	273	46			
F	370	370	29	-	3	20	34	1	5	34	19	41	2	49	81	41	5	0	2	2432	2395	95	3	356	393	48				
B	34	111	38	-	10	42	42	4	2	236	235	44	8	7	276	250	100	4	6	3	16	51	35	4	209	181	47			
9	430	412	45	-	11	I31	894	56	5	41	34	43	5	202	233	42	8	70	53	26	5	63	110	37	6	1080	1060	66		
10	809	795	51	-	12	277	311	44	5	41	34	43	5	202	233	42	9	1888	1775	68	6	1345	1322	69	7	110	117	66		
11	64	108	34	-	13	313	381	49	***	4	K	0	***	6	45	49	40	10	37	30	11	3	3	41	-					
13	0	0	-	***	3	K	5	***	E	7669	6897	207	8	193	201	39	12	23	0	30	***	6	K	-3	***					
***	3	K	-1	***	1	307	252	41	1	189	205	23	9	727	743	51	13	27	27	42	***	5	K	-5	***					
1	5	65	24	-	3	1160	1098	50	2	51	43	20	10	156	168	41	14	13	0	42	16	179	167	99	1	526	532	44		
2	452	480	24	-	5	192	183	42	5	166	137	41	***	4	K	-4	***	5	177	83	44	5	1	0	41					
3	4	0	16	-	6	54	62	69	35	6	438	422	35	0	248	311	40	4	232	233	45	5	221	229	40	***	6	K	4	***
N	534	480	27	-	7	468	509	45	7	3	15	26	0	1	284	2662	100	1	531	497	60	6	585	562	48	0	1	0	37	
E	6	160	165	-	8	255	292	32	8	936	851	43	2	254	251	50	3	792	775	57	8	157	192	44	***	6	K	4	***	
6	528	537	10	-	9	291	233	40	10	199	187	28	3	321	374	42	5	151	121	49	***	5	K	-1	***					
8	108	162	38	-	11	199	196	42	11	20	11	25	4	5	30	51	36	4	232	233	45	5	221	229	40	***	6	K	4	***
10	678	689	48	-	12	576	579	51	12	353	342	45	5	350	350	40	6	118	121	49	***	5	K	-1	***					
11	318	343	41	***	3	K	-5	***	14	0	46	42	10	5	101	105	45	8	81	85	33	5	105	88	43	0	234	272	41	
12	202	202	61	-	15	37	37	1	10	409	446	46	10	159	207	42	11	15	20	44	5	385	384	48	***	6	K	-4	***	
13	5	K	2	***	4	810	843	53	***	4	K	1	***	5	K	-1	***	6	1655	1716	82	0	1360	1379	71	1	27	91	39	
E	0	5635	4798	150	-	6	449	387	43	1	62	64	22	1	4	11	36	3	17	19	39	***	5	K	-6	***				
1	16	45	16	-	7	91	135	41	2	263	255	26	1	4	11	36	3	2	3010	2878	108	3	213	168	41					
2	560	500	27	-	8	148	108	37	3	620	591	33	2	51	73	30	3	274	321	45	4	2663	2624	102	3	41	97	39		
E	3	2334	2275	76	-	9	119	114	40	4	248	247	39	3	387	339	46	5	488	514	50	4	270	264	44	0	244	244	44	
E	4	5350	4700	148	-	10	129	139	39	5	178	177	40	4	84	86	38	6	557	562	60	5	446	446	44	0	244	244	44	
6	518	518	41	-	11	1181	1158	64	E	5176	2000	200	6	1	523	564	29	7	17	175	108	1	1	130	1379	71	1	27	91	39
6	560	467	40	-	12	30	71	42	7	308	386	45	6	820	746	51	8	2014	1881	82	2	173	168	41	0	224	216	44		
7	62	65	35	-	8	106	111	34	9	16	11	34	7	1553	1367	67	9	767	765	50	4	447	533	46	0	211	197	46		
8	53	135	43	***	3	K	6	***	9	65	77	33	8	32	61	36	8	338	388	45	5	536	505	48	***	6	K	5	***	
9	1228	1202	61	-	10	446	401	44	11	464	401	44	9	446	432	46	6	1156	1126	63	4	162	167	42	1	162	167	42		
10	788	780	50	-	11	101	101	50	11	684	759	53	10	374	385	47	11	123	113	43	***	5	K	7	***					
11	111	111	50	-	12	51	25	39	0	350	300	55	9	256	277	44	12	100	99	37	3	258	279	50	0	160	222	48		
12	1155	1034	42	***	5	122	118	37	13	523	564	29	1	523	564	29	13	203	198	50	0	155	23	30	0	1054	1056	66		
13	1156	1072	78	-	14	162	161	37	15	208	210	30	12	503	536	47	14	102	156	61	5	523	555	56	0	101	101	56		
E	0	1329	1151	51	-	16	43	43	17	201	201	37	13	523	564	29	15	201	201	82	1	157	168	41	0	130	131	75		
E	2	1561	1488	52	-	18	6	15	19	5	1941	1847	78	6	80	98	30	16	557	676	51	1	90	128	46	0	224	216	44	
A	4	558	520	42	***	3	K	-6	***	19	230	250	42	8	652	732	54	20	446	446	42	0	117	117	42	0	224	216	44	
5	1588	1320	42	***	6	99	98	32	7	59	70	40	10	256	277	44	21	102	131	42	0	157	168	41	0	224	216	44		
6	96	93	32	-	8	152	143	38	11	115	104	38	12	51	56	40	11	403	436	44	1	152	151	41	0	155	156	42		
7	467	467	49	-	9	123	117	37	12	96	100	40	13	51	56	40	12	403	436	44	1	152	151	41	0	155	156	42		
8	467	467	49	***	9	208	210	30	14	43	43	32	14	51	56	40	14	201	201	82	1	152	151	41	0	155	156	42		
9	375	366	57	-	10	241	243	33	9	895	870	55	15	163	176	38	15	397	432	44	2	151	151	41	0	152	151	41		
10	451	451	31	-	11	969	998	55	10	550	557	56	16	163	176	38	16	255	254	50	3	159	158	41	0	152	151	41		
11	1453	1521	69	-	12	108	138	37	11	38	107	38	1	12	125	187	43	17	505	531	42	7	10	121	41	0	152	151	41	
12	696	696	51	-	13	581	819	52	12	250	250	55	18	123	158	36	18	505	531	42	7	10	121	41	0	152	151	41		
13	1778	1744	80	-	14	1786	1657	72	14	277	291	44	19	1	696	746	50	19	346	375	35									

Table 3. When converted to Å, the standard errors of the coordinates amount to about 0.001 Å for the niobium atom and 0.002 to 0.003 Å for the fluorine and potassium atoms.

Table 4 presents the values F_o^2 , σ_{F^2} , and F_c^2 , with the sign of F_c , for all of the observations.

A composite three-dimensional Fourier map is shown in Fig. 2. The F_o coefficients used in the synthesis were obtained from the set of F_o^2 data corrected for extinction that was used in refinement F . The signs were from the F_c values computed with the use of the parameters in Table 3. A final three-dimensional difference map showed only random background ripples. The greatest excursion of density on each side of zero was in magnitude only about 2% of the peak density of the niobium atom.

Structure and thermal motion

Structure

An understanding of the sense in which our results confirm those of Hoard may be reached quickly by viewing the shifts of atomic positions between the two structures (Table 5). The shifts are small in the sense that the basic geometric description of the crystal structure remains unchanged, including the description of the distorted NbF_7^{2-} anion. On the other hand, the shifts are all large relative to the standard errors. The direction of the shifts of the various atoms vary widely. The two largest shifts, those of F(3) and F(6), are at right angles.

General features of the crystal structure are shown in the stereoscopic view of Fig. 3. This view, showing a

Table 5. Shifts of atomic positions in the refinement of K_2NbF_7

Atom	Shift	Atom	Shift
Nb	0.036 Å	F(3)	0.106 Å
K(1)	0.046	F(4)	0.030
K(2)	0.066	F(5)	0.074
F(1)	0.026	F(6)	0.154
F(2)	0.060	F(7)	0.088

ball-and-stick model for the complex ion NbF_7^{2-} , complements the packing drawing shown by Hoard in his Fig. 3(b).*

Details of the structure of the complex ion NbF_7^{2-} are shown in Fig. 4, where Nb–F bond lengths and F–F neighbor distances are given, and in Table 6, where F–Nb–F angles are given.† The first value in Fig. 4 for each Nb–F bond length is the value of the internuclear separation computed directly from the coordinates of Table 3. The second value, in parentheses, has been corrected for the effects of thermal motion by the method of Busing & Levy (1964), assuming the so-called ‘riding’ model (see the discussion of thermal motion below). The range of Nb–F bond lengths after correction is 1.940–1.978 Å, only half as wide as the range 1.911–1.971 Å before correction.

The widely varying intraionic F–F neighbor distances (2.357–2.912 Å) and F–Nb–F valence angles

* In comparing our Fig. 3 with Hoard’s Fig. 3(b), one should note that the latter drawing is based on a left-handed coordinate system.

† The program of Busing, Martin & Levy (1964) was used in calculating distances and angles and in analyzing the thermal parameters. The least-squares covariance matrix was used in calculating standard errors.

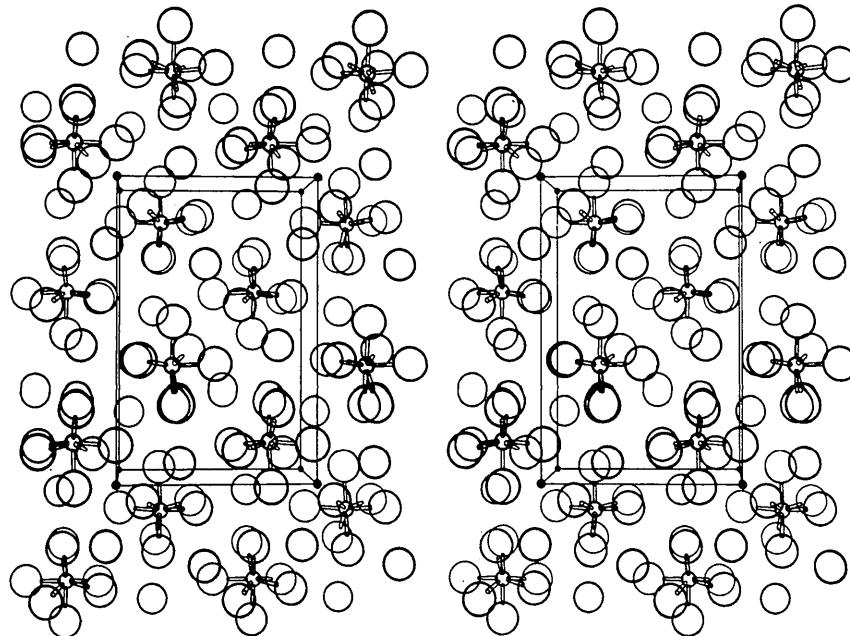


Fig. 3. Stereoscopic view of the K_2NbF_7 crystal structure. The direction of view is that of the vector $-\mathbf{a}$. The axis \mathbf{b} is vertical from top to bottom and the axis \mathbf{c} is horizontal from left to right in the view. The reference asymmetric unit is the one within the cell outline at the upper left.

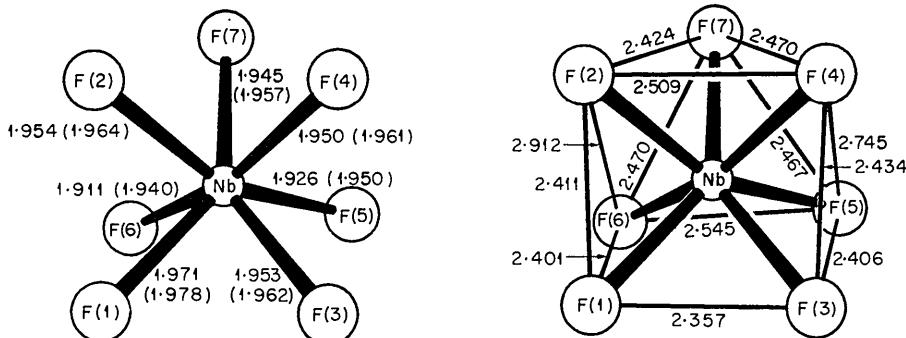


Fig. 4. Interatomic distances (\AA) in the NbF_7^{2-} ion. The Nb-F distances in parentheses have been corrected for thermal motion ('riding' model). The computed standard errors are 0.002 \AA to 0.003 \AA for the uncorrected Nb-F bond lengths and 0.003 \AA to 0.004 \AA for the F-F distances. The corrected bond lengths are thought to be accurate within 0.01 \AA .

(73.86° - 97.74°) describe the irregular shape of the ion. The irregularities are, presumably, the result of electrostatic interactions between the fluorine atoms in the anion and the potassium ions which surround it. Fig. 5 shows the arrangement of the potassium ions about

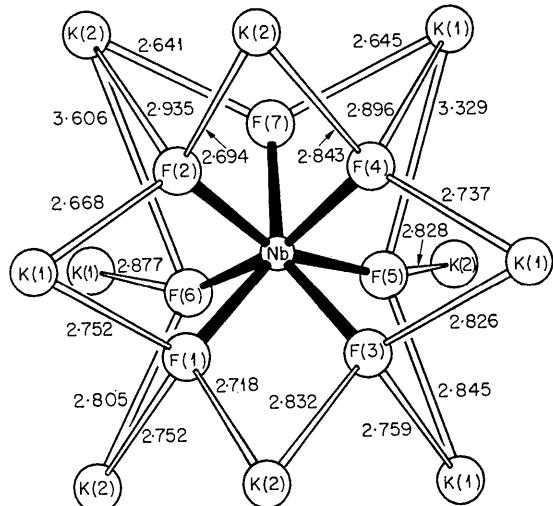


Fig. 5. The NbF_7^{2-} ion and surrounding K^+ ions in the K_2NbF_7 structure (parallel projection). The computed standard errors of the K-F distances are 0.003 \AA to 0.004 \AA .

the anion, with all of the K-F near-neighbor distances given. Atoms F(1), F(2), F(3), and F(4) are each in contact with three potassium ions in an approximately trigonal arrangement about the axis through niobium and fluorine. Atoms F(5), F(6), and F(7) are each in close contact with only two potassium ions, although F(5) and F(6) each have an additional potassium neighbor at a distance distinctly greater than the average K-F distance. The shortest three of the seven Nb-F bonds are those involving these fluorine atoms having only two potassium near neighbors.

Table 6. Angles $F(i)$ -Nb- $F(j)$ in the NbF_7^{2-} ion
The standard errors of these angles are all about 0.1°

<i>i</i>	<i>j</i>	Angle
7	2	76.9°
7	4	78.7
7	5	79.2
7	6	79.7
2	6	97.7
4	5	90.2
2	4	80.0
5	6	83.1
1	2	75.8
1	6	76.4
3	4	77.2
3	5	76.7
1	3	73.9

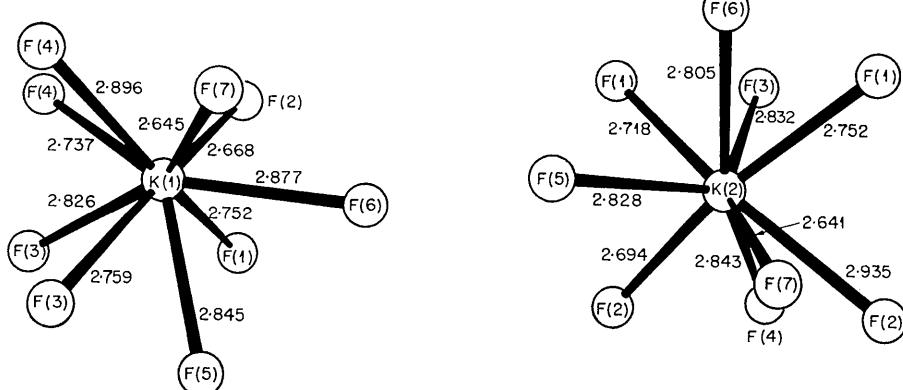


Fig. 6. Perspective views showing the coordination about K(1) and K(2) in K_2NbF_7 .

The ninefold coordination about each of the two potassium ions (Fig. 6) may be conceived as derived by considerable distortion from an arrangement in which eight fluorine atoms are at the corners of a cube and a ninth atom projects from one cube face. The range of K–F distances is much the same for the two potassium ions.

The shortest interionic F–F distances are: 3.07 Å between F(2) and F(4) in glide-related ions; 3.08 Å between F(7) and F(7) centrosymmetrically related (compare with Hoard's distance 2.91 Å); and 3.18 Å between F(6) and F(7) in centrosymmetrically related ions). There are many F–F distances near 3.30 Å. The range of F–F distances is quite normal.

Thermal motion

Principal-axis transformations have been performed on the thermal-parameter ellipsoids of the ten atoms of the asymmetric unit. The root-mean-square axial displacements and the axial orientations are given in Table 7. The thermal motion of the NbF_7^{2-} ion and the 10 K^+ ions which surround it can be visualized readily from the stereoscopic drawing of Fig. 7, in which the direction of view is almost the same as that in Figs. 4 and 5. In the drawing the thermal motion of each atom is represented by a thermal-motion probability ellipsoid (Johnson, 1965) centered at the mean position of the

atom. The lengths of the principal axes represent the r.m.s. vibrational displacements in the principal-axis directions on a scale 2.25 times that of the internuclear separations. The ellipsoids are, therefore, the 83% probability ellipsoids.

Originally we had supposed that the thermal parameters of the atoms of the NbF_7^{2-} ion would likely be interpretable readily in terms of rigid-body motion of the ion. One can see, however, merely from inspection of Fig. 7 that the large r.m.s. displacements of F(5) and F(6) are inconsistent with the rigid-body model. Detailed analysis (Cruickshank, 1956) does show that the thermal parameters of the other six atoms describe fairly well the motion of a rigid structural unit. The analysis is of doubtful utility, however, because its ultimate purpose, the correction of bond lengths, can be achieved in a more direct manner in this particular case through use of the 'riding' model (Busing & Levy, 1964).

That the riding model is appropriate for calculation of corrections to the Nb–F bond lengths is a direct conclusion from consideration of data in Table 7. The motion of the niobium atom is almost exactly isotropic. For each fluorine atom F(i) the principal axis along which the r.m.s. displacement is least is nearly parallel (within 8°) to the corresponding vector $\text{Nb} \rightarrow \text{F}(i)$. The r.m.s. displacement along this axis is in each case very

Table 7. Description of atomic thermal-parameter ellipsoids: orientations of the principal axes and root-mean-square axial displacements

The orientation of each principal axis is specified with respect to the crystal axial vectors \mathbf{a} , \mathbf{b} , \mathbf{c} and, for each fluorine atom F(i), with respect to the vector $\text{Nb} \rightarrow \text{F}(i)$. Standard errors appear in parentheses.

Atom	Principal axis	R.m.s. displacement	Angle (°) to reference axis			
			a	b	c	$\text{Nb} \rightarrow \text{F}(i)$
Nb	1	0.110(3) Å	77(15)	141(13)	54(9)	—
	2	0.116(3)	37(15)	61(17)	70(15)	—
	3	0.123(3)	56(14)	113(10)	137(9)	—
K(1)	1	0.134(4)	81(5)	161(6)	107(7)	—
	2	0.162(4)	97(17)	108(7)	19(10)	—
	3	0.173(4)	11(11)	84(7)	81(16)	—
K(2)	1	0.140(5)	88(17)	110(3)	160(4)	—
	2	0.153(5)	178(15)	90(7)	92(16)	—
	3	0.191(4)	91(5)	160(3)	70(3)	—
F(1)	1	0.125(3)	40(3)	130(3)	85(3)	172(3)
	2	0.165(3)	53(15)	41(11)	75(52)	83(4)
	3	0.168(3)	77(33)	82(40)	165(50)	87(7)
F(2)	1	0.126(3)	131(6)	139(6)	90(3)	172(6)
	2	0.146(3)	135(6)	52(6)	111(2)	82(6)
	3	0.214(3)	75(2)	104(2)	159(2)	90(1)
F(3)	1	0.125(3)	44(2)	76(3)	130(2)	172(2)
	2	0.172(3)	113(16)	24(20)	95(19)	86(4)
	3	0.177(3)	125(12)	108(24)	139(4)	96(3)
F(4)	1	0.120(4)	123(4)	81(2)	145(3)	178(3)
	2	0.155(3)	145(3)	108(2)	61(4)	91(4)
	3	0.217(3)	80(2)	160(2)	107(1)	92(1)
F(5)	1	0.122(3)	102(2)	166(1)	96(2)	5(1)
	2	0.175(3)	138(1)	85(3)	49(1)	89(3)
	3	0.296(3)	129(1)	77(1)	138(1)	95(1)
F(6)	1	0.132(3)	104(3)	97(3)	165(1)	5(3)
	2	0.174(3)	135(1)	46(1)	85(4)	86(4)
	3	0.328(3)	132(1)	135(1)	75(1)	92(1)
F(7)	1	0.132(3)	30(2)	106(3)	115(2)	4(3)
	2	0.181(3)	90(4)	32(5)	122(5)	94(3)
	3	0.201(3)	120(2)	117(6)	137(5)	89(2)

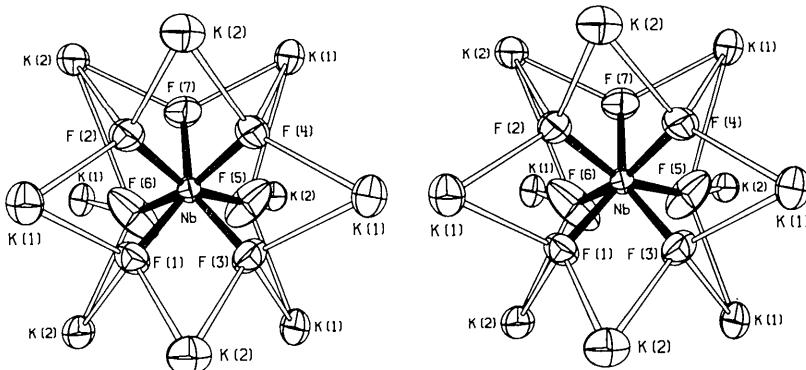


Fig. 7. Stereoscopic view showing thermal-motion probability ellipsoids of the NbF_7^{2-} ion and the surrounding K^+ ions.

close in magnitude to the isotropic displacement of the niobium atom. The two larger displacements of each atom $F(i)$ are nearly perpendicular to the $\text{Nb} \rightarrow F(i)$ vector. Evidently, the motion of each fluorine atom is to a close approximation a librational motion about the central niobium atom compounded with the translatory vibrational motion of the niobium atom. Thus, the NbF_7^{2-} ion is an almost ideal case for use of the riding model in computing corrections to bond lengths.

The exceptionally large vibrational displacements of $F(5)$ and $F(6)$ seem plausible in view of the packing about these atoms. As already noted, the arrangement of potassium ions about $F(5)$ and $F(6)$ is different from that characteristic of most of the other fluorine atoms. The large displacement, for $F(5)$ and for $F(6)$, is nearly perpendicular to the plane of the fluorine atom and its two close potassium neighbors. The relation of two $F(7)$ atoms across a symmetry center at the unusually short distance 3.08 \AA is just such as to inhibit the similar large displacement that might be expected for atom $F(7)$.

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